

Chapter 9. Impulse and Momentum

Explosions and collisions obey some surprisingly simple laws that make problem solving easier when comparing the situation *before* and *after* an interaction.

Chapter Goal: To introduce the ideas of impulse and momentum and to learn a new problem-solving strategy based on conservation laws.

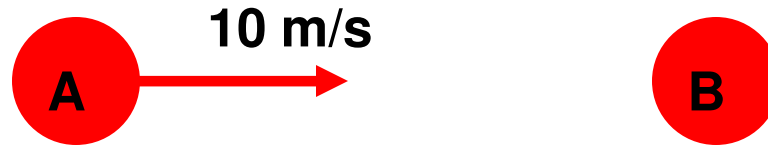


Chapter 9. Impulse and Momentum

Topics:

- Momentum and Impulse
- Solving Impulse and Momentum Problems
- Conservation of Momentum
- Inelastic Collisions
- Explosions
- Momentum in Two Dimensions

Momentum



After the collision



What is the velocity of ball A after the collision? ball B?

What is conserved during the collision?

MOMENTUM

$$\vec{p} = m\vec{v}$$

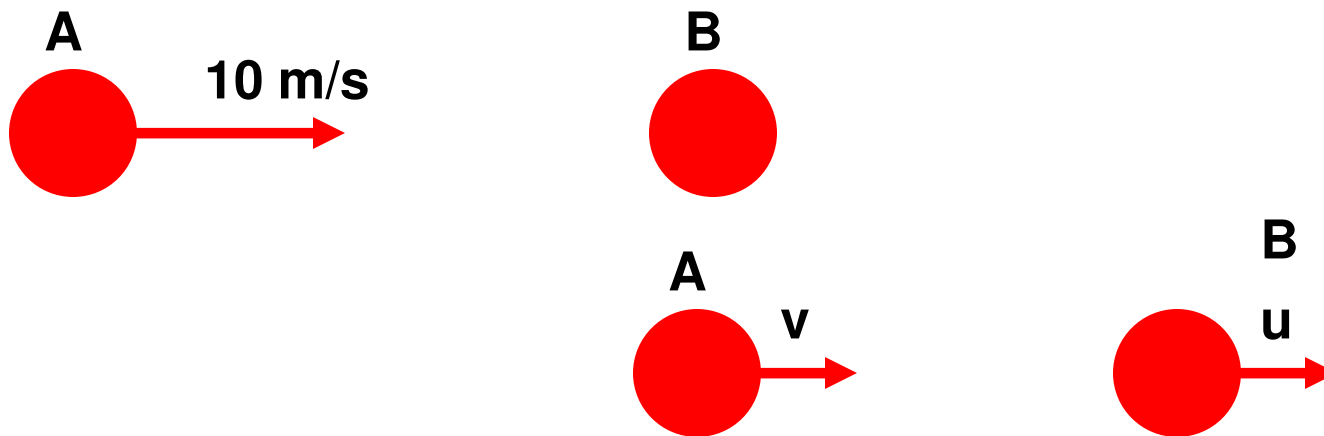
The total momentum is the sum of momentum of ball A and momentum of ball B.

Momentum

$$\vec{p} = m\vec{v}$$

The total momentum of the system is conserved during the collision:

$$m_A v_{A,i} = m_A v + m_B u$$

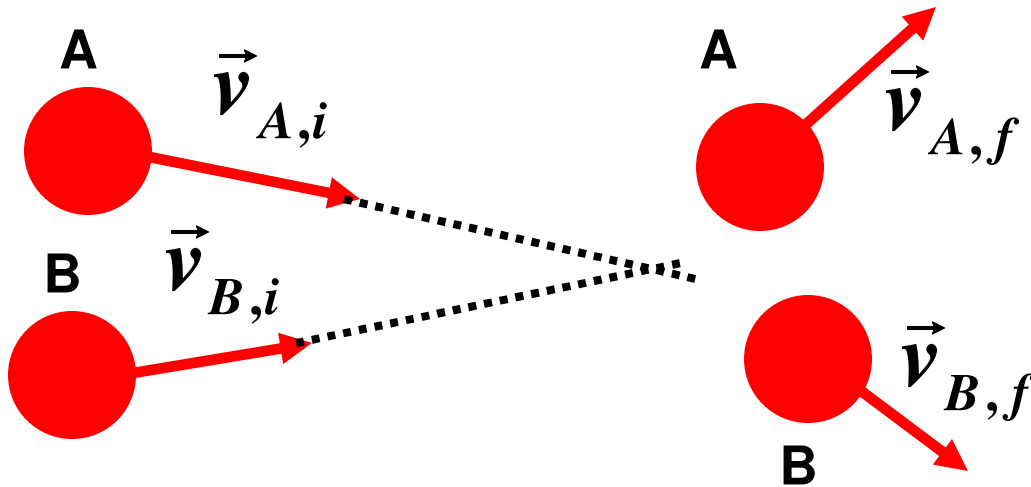


- Momentum is a vector. It has the same direction as corresponding velocity.
- General expression for the momentum conservation: the total momentum before the collision is equal to the total momentum after the collision

Momentum

$$\vec{p} = m\vec{v}$$

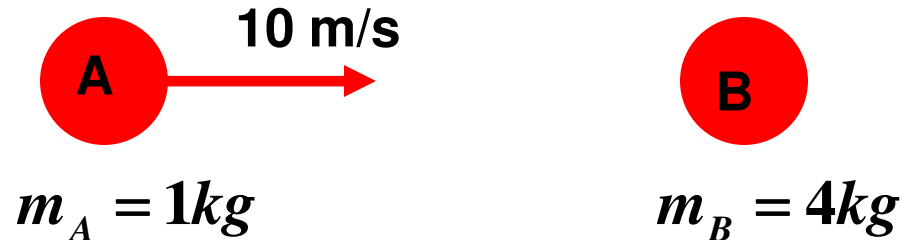
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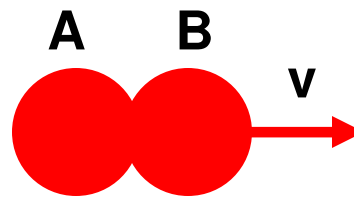
$$m_A \vec{v}_{A,i} + m_B \vec{v}_{B,i} = m_A \vec{v}_{A,f} + m_B \vec{v}_{B,f}$$

Usually this equation is written in terms of components.

Example:



After the collision the balls are moving together (have the same velocity). What is their velocity?



Momentum before the collision: $p_i = m_A v_{A,i} = 10 \frac{\text{kg} \cdot \text{m}}{\text{s}}$

Momentum after the collision: $p_f = (m_A + m_B)v = 5v$

Conservation of momentum: $p_i = p_f$

$$10 = 5v \quad v = 2\text{m / s}$$

Why do we have conservation of total momentum?

Newton's second law:

$$\vec{F}_{net} = m\vec{a}$$

Acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Then

$$\vec{F}_{net} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

momentum



After integration

$$\Delta\vec{p} = \int_{t_1}^{t_2} \vec{F}_{net} dt$$

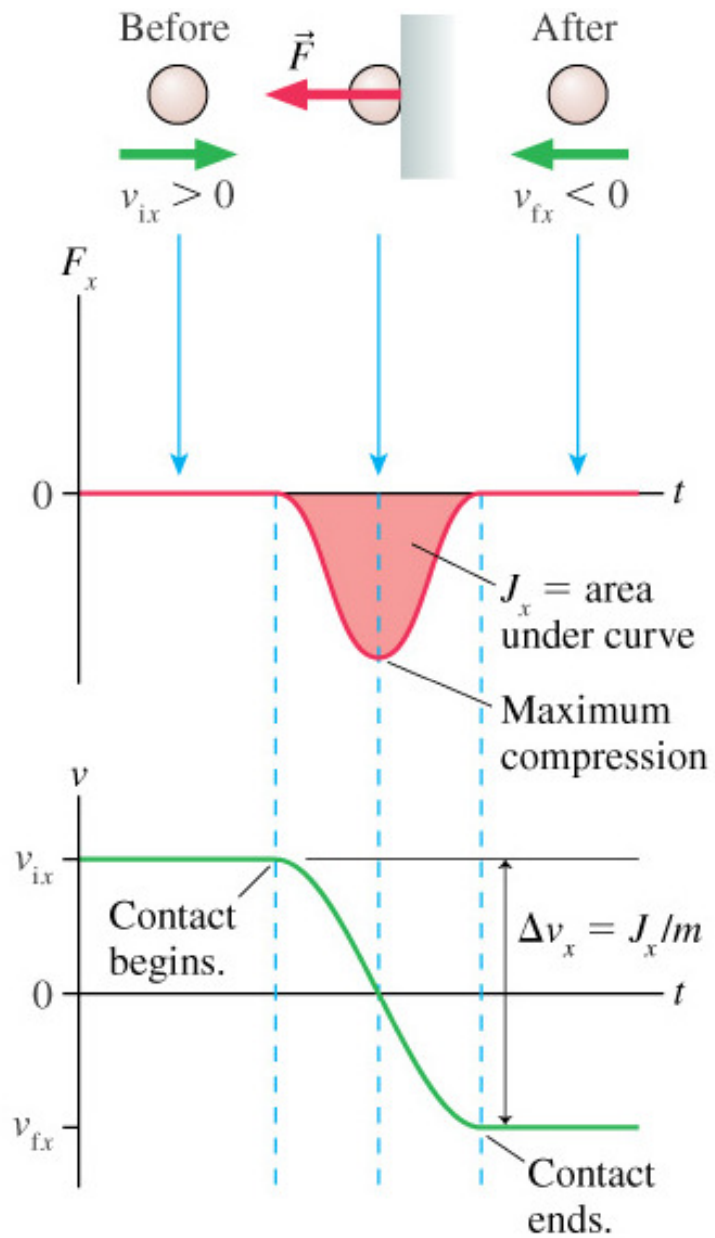
The area under $\vec{F}_{net}(t)$ curve.

It is called **IMPULSE**, \vec{J} .

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{net} dt$$

The impulse of the force is equal to the change of the momentum of the object.

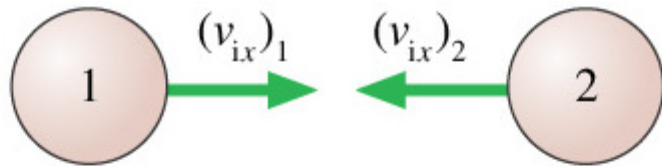
$$\Delta\vec{p} = \vec{J}$$



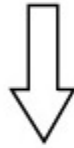
$$p_i = mv_{ix}$$

$$p_f = mv_{fx} < 0$$

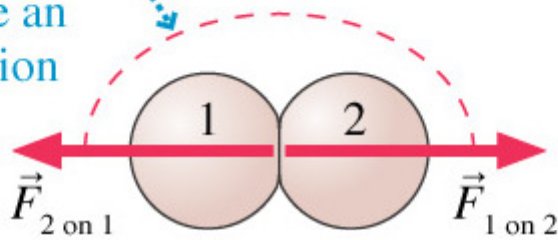
$$J_x = p_f - p_i < 0$$



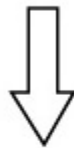
Before



The forces during the collision are an action/reaction pair.



During



After

$$m_1 v_{fx,1} - m_1 v_{ix,1} = \int_{t_1}^{t_2} F_{x,2on1} dt$$

$$m_2 v_{fx,2} - m_2 v_{ix,2} = \int_{t_1}^{t_2} F_{x,1on2} dt$$

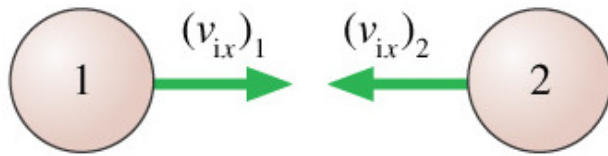
Newton's third law:

$$F_{x,1on2} = -F_{x,2on1}$$

Then

$$\int_{t_1}^{t_2} F_{x,1on2} dt = -\int_{t_1}^{t_2} F_{x,2on1} dt$$

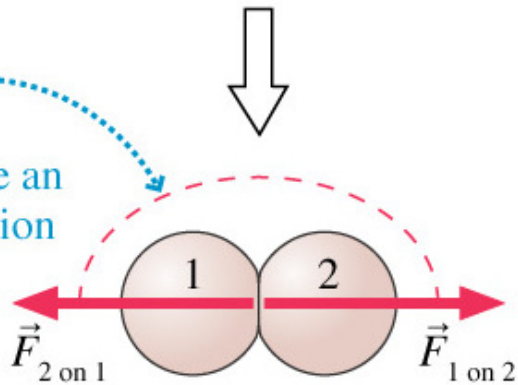
$$m_2 v_{fx,2} - m_2 v_{ix,2} = -\left(m_1 v_{fx,1} - m_1 v_{ix,1}\right)$$



Before

$$m_2 v_{fx,2} - m_2 v_{ix,2} = - (m_1 v_{fx,1} - m_1 v_{ix,1})$$

The forces during the collision are an action/reaction pair.



During

$$m_1 v_{ix,1} + m_2 v_{ix,2} = m_1 v_{fx,1} + m_2 v_{fx,2}$$

$$p_{ix,1} + p_{ix,2} = p_{fx,1} + p_{fx,2}$$



After

$$p_{ix,total} = p_{fx,total}$$

The law of conservation of momentum

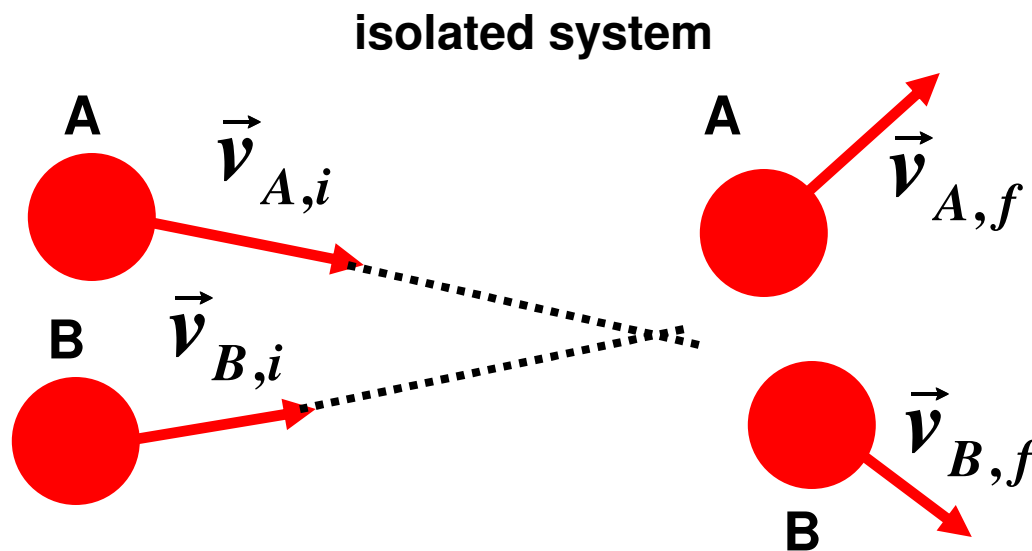
Momentum

$$\vec{p} = m\vec{v}$$

The law of conservation of momentum:

The total momentum of an isolated system (no external forces) does not change.

Interactions within system do not change the system's total momentum



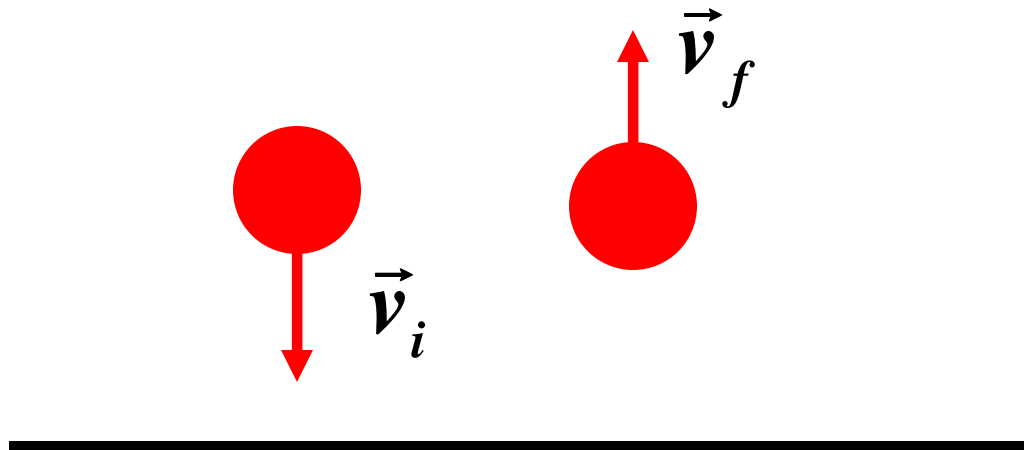
$$m_A \vec{v}_{A,i} + m_B \vec{v}_{B,i} = m_A \vec{v}_{A,f} + m_B \vec{v}_{B,f}$$

Momentum

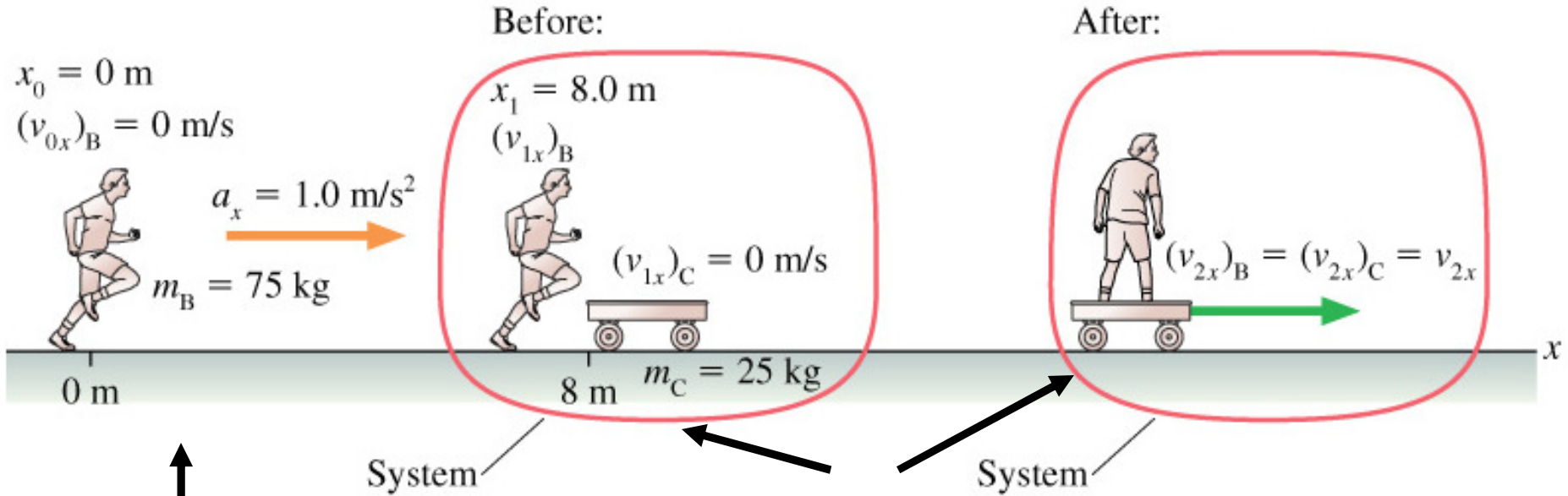
$$\vec{p} = m\vec{v}$$

The ball is dropped onto a hard floor:

- The ball is not an isolated system (interaction with the floor)
- no conservation of momentum for the ball
- Initial momentum is $\vec{p}_i = m\vec{v}_i$
- Final momentum (after collision) is $\vec{p}_f = m\vec{v}_f$
- The ball+ the floor is an isolated system
- The total momentum (ball+floor) is conserved



Example: Find v_{2x}



Motion with constant acceleration:

$$(v_{1x,B})^2 = 2a_x x_1 = 16$$

$$v_{1x,B} = 4 \text{ m/s}$$

Momentum before the "collision":

$$p_{i,total} = m_B v_{1x,B} + m_C v_{1x,C} = m_B v_{1x,B} = 75 \cdot 4 = 300 \frac{\text{kg} \cdot \text{m}}{\text{s}}$$

Momentum after the "collision":

$$p_{f,total} = m_B v_{2x,B} + m_C v_{2x,C} = (m_B + m_C) v_{2x} = 100 v_{2x}$$

Conservation of momentum:

$$p_{f,total} = p_{i,total}$$

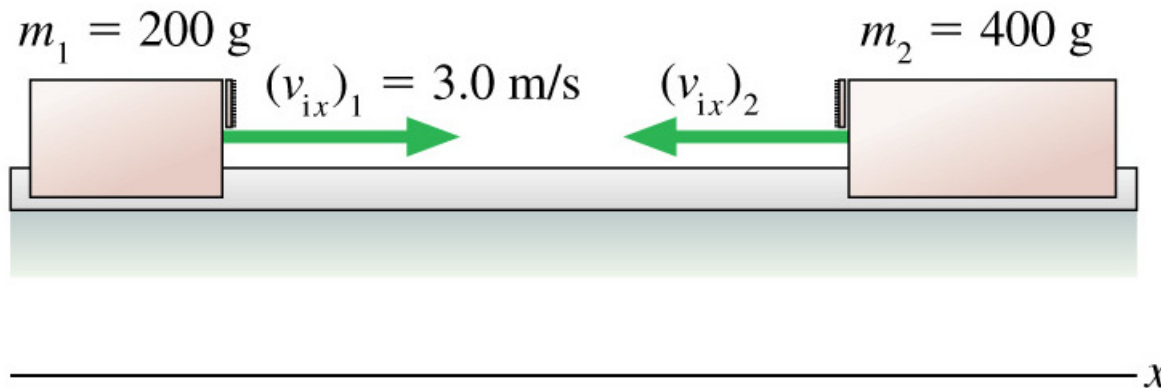
$$100 v_{2x} = 300$$

$$v_{2x} = 3 \text{ m/s}$$

Perfectly inelastic collision:

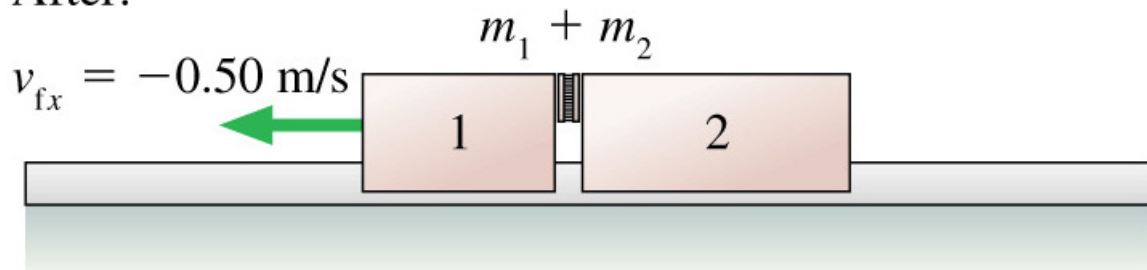
A collision in which the two objects stick together and move with a common final velocity.

Before:



$$p_{i,total} = m_1 v_{ix,1} + m_2 v_{ix,2}$$

After:



$$p_{f,total} = (m_1 + m_2) v_{fx}$$

Find: $(v_{ix})_2$

$$m_1 v_{ix,1} + m_2 v_{ix,2} = (m_1 + m_2) v_{fx}$$

$$v_{ix,2} = \left(\frac{m_1}{m_2} + 1 \right) v_{fx} - \frac{m_1}{m_2} v_{ix,1} = -\frac{3}{2} \cdot 0.5 - \frac{3}{2} = -2.25 \text{ m/s}$$

Chapter 9. Summary Slides

General Principles

Law of Conservation of Momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \dots$ of an isolated system is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Newton's Second Law

In terms of momentum, Newton's second law is

$$\vec{F} = \frac{d\vec{p}}{dt}$$

General Principles

Solving Momentum Conservation Problems

MODEL Choose an isolated system or a system that is isolated during at least part of the problem.

VISUALIZE Draw a pictorial representation of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{fx})_1 + (p_{fx})_2 + \cdots = (p_{ix})_1 + (p_{ix})_2 + \cdots$$

$$(p_{fy})_1 + (p_{fy})_2 + \cdots = (p_{iy})_1 + (p_{iy})_2 + \cdots$$

ASSESS Is the result reasonable?

Important Concepts

Momentum $\vec{p} = m\vec{v}$

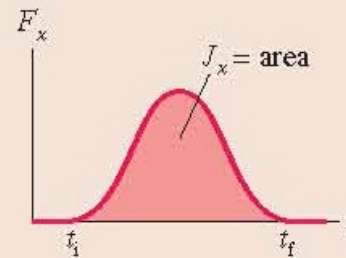


Impulse $J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve}$

Impulse and momentum are related by the **impulse-momentum theorem**

$$\Delta p_x = J_x$$

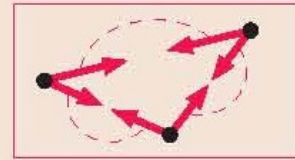
This is an alternative statement of Newton's second law.



Important Concepts

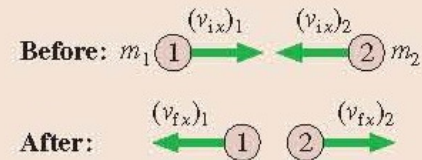
System A group of interacting particles.

Isolated system A system on which there are no external forces or the net external force is zero.



Before-and-after pictorial representation

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.

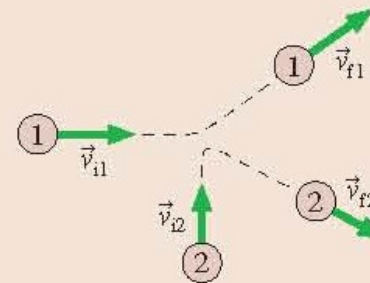


Applications

Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.

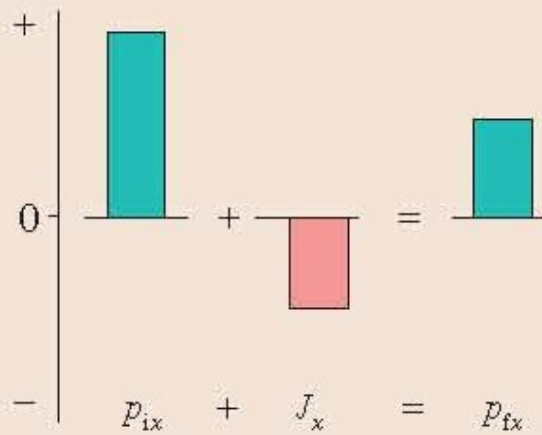
Explosions Two or more particles move away from each other.

Two dimensions No new ideas, but both the x - and y -components of P must be conserved, giving two simultaneous equations.



Applications

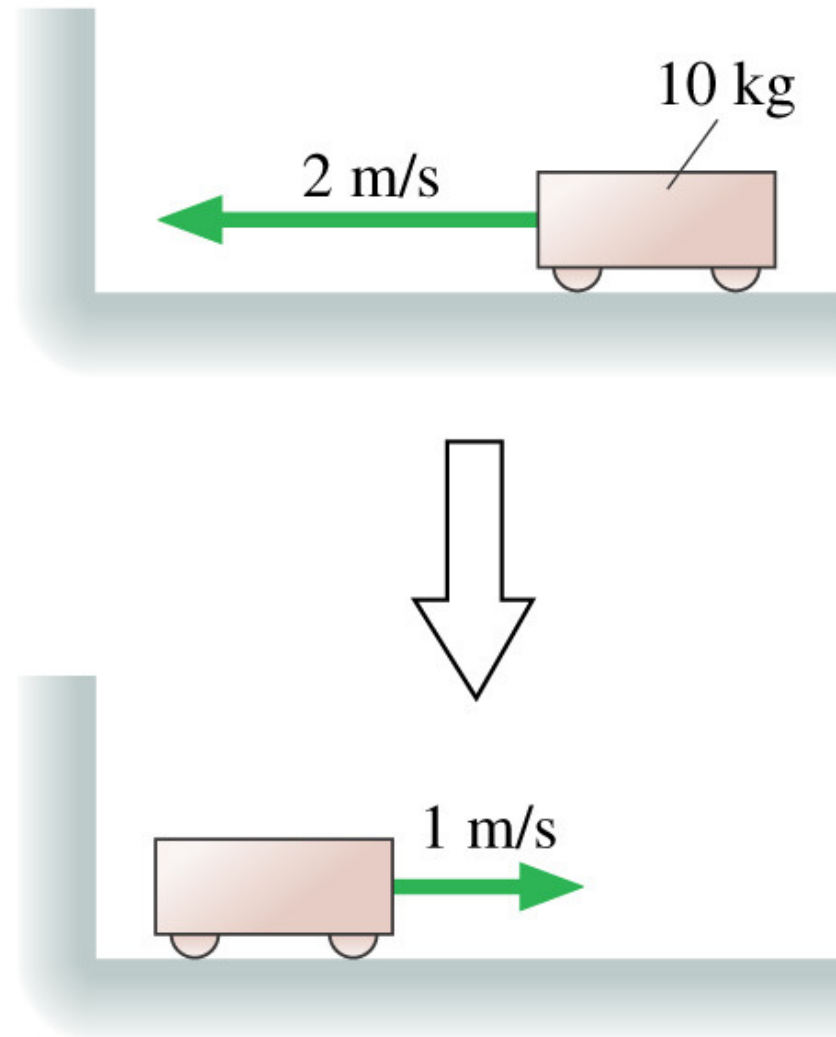
Momentum bar charts display the impulse-momentum theorem $p_{fx} = p_{ix} + J_x$ in graphical form.



Chapter 9. Questions

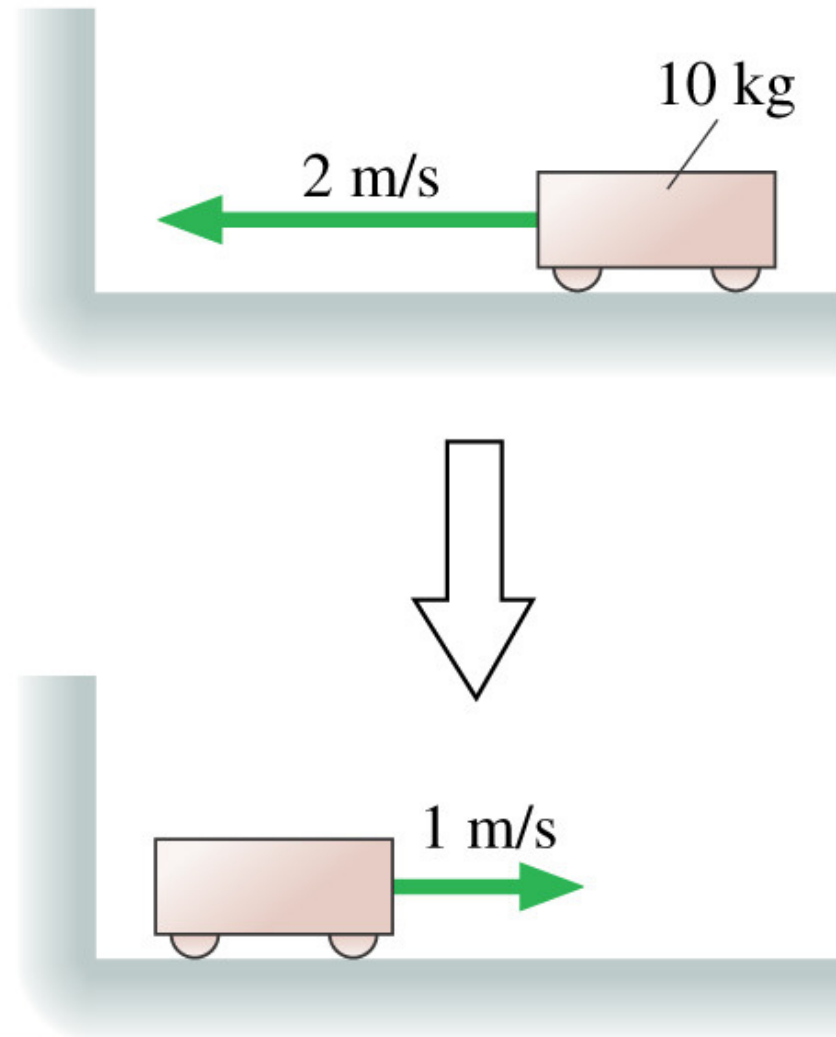
The cart's change of momentum is

- A. 30 kg m/s.
- B. 10 kg m/s.
- C. -10 kg m/s.
- D. -20 kg m/s.
- E. -30 kg m/s.



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A 10 g rubber ball and a 10 g clay ball are thrown at a wall with equal speeds. The rubber ball bounces, the clay ball sticks. Which ball exerts a larger impulse on the wall?

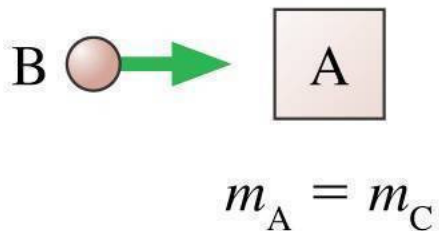
- A. They exert equal impulses because they have equal momenta.
- B. The clay ball exerts a larger impulse because it sticks.
- C. Neither exerts an impulse on the wall because the wall doesn't move.
- D. The rubber ball exerts a larger impulse because it bounces.

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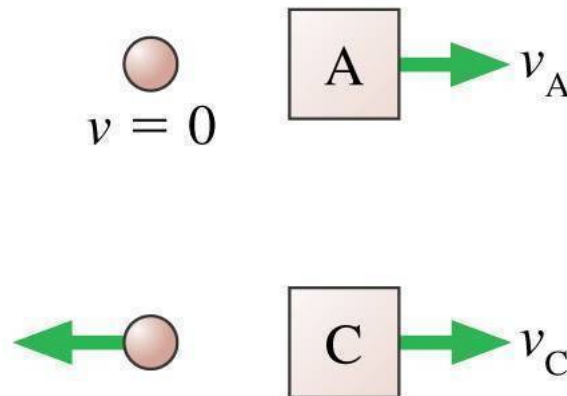
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Objects A and C are made of different materials, with different “springiness,” but they have the same mass and are initially at rest. When ball B collides with object A, the ball ends up at rest. When ball B is thrown with the same speed and collides with object C, the ball rebounds to the left. Compare the velocities of A and C after the collisions. Is v_A greater than, equal to, or less than v_C ?

Before



After



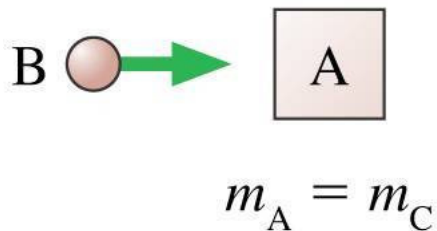
A. $v_A > v_C$

B. $v_A < v_C$

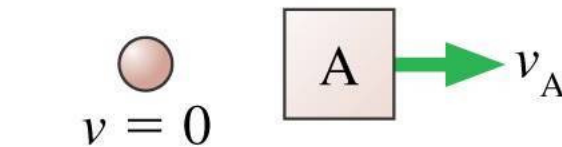
C. $v_A = v_C$

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Before



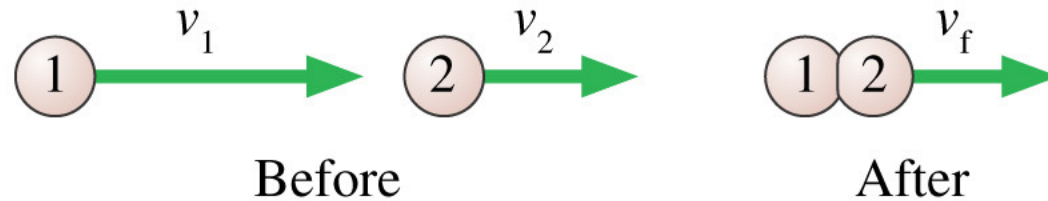
After



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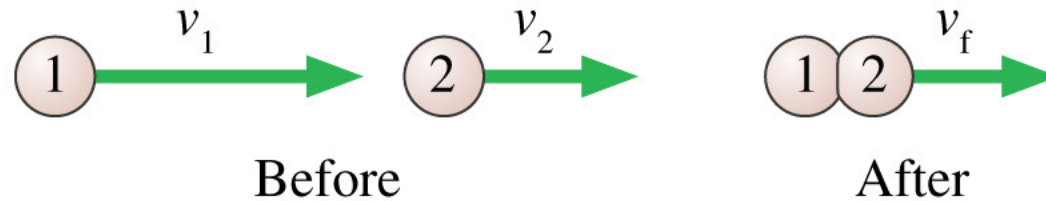
B. $v_A < v_C$

C. $v_A = v_C$



The two particles are both moving to the right. Particle 1 catches up with particle 2 and collides with it. The particles stick together and continue on with velocity v_f . Which of these statements is true?

- A. $v_f = v_2$.
- B. v_f is less than v_2 .
- C. v_f is greater than v_2 , but less than v_1 .
- D. $v_f = v_1$.
- E. v_f is greater than v_1 .



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An explosion in a rigid pipe shoots out three pieces. A 6 g piece comes out the right end. A 4 g piece comes out the left end with twice the speed of the 6 g piece. From which end does the third piece emerge?

- A. Right end
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